

B.Sc. Semester – 1 (NEP-2020) Examination

January-2024

Calculus and Solid Geometry (Major-2)

Time: 2:00 Hours

Marks: 50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one.

- (1) State and prove Lagrange's mean value theorem. (05)
- (2) Obtain expansion of $\log x$ in ascending power of $x - 1$. (05)

Que.1(B) Answer any one.

- (1) Derive power series expansion of $f(x) = \sin x$ in terms of x . (05)
 - (2) Verify Cauchy's mean value theorem for the function (05)
- $$f(x) = \sin x, g(x) = \cos x, \forall x \in \left[0, \frac{\pi}{2}\right].$$

Que.2(A) Answer any one.

- (1) State and prove L'Hospital's rule for $\frac{0}{0}$ form. (05)
- (2) Evaluate: $\lim_{x \rightarrow 0} \left(\cot^2 x - \frac{1}{x^2} \right)$. (05)

Que.2(B) Answer any one.

- (1) If $\lim_{x \rightarrow 0} \frac{\sin 2x + a \sin x}{x^3} = b$ then find the value of a and b . (05)
- (2) Evaluate: $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2 \log(1+x)}{x \sin x}$ (05)

Que.3(A) Answer any one.

- (1) Derive the equation of tangent plane at any point (α, β, γ) of the sphere (05)
- $$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0.$$
- (2) Define: Sphere. Find the equation of sphere through the four points (05)
- $$(4, -1, 2), (0, -2, 3), (1, 5, -1), (2, 0, 1).$$

Que.3(B) Answer any one.

- (1) Find the equation of spheres which pass through the circle (05)
- $$x^2 + y^2 + z^2 = 5, x + 2y + 3z = 3 \text{ and touch the plane } 4x + 3y = 15.$$
- (2) Find the area of circle $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$, (05)
- $$x - 2y + 2z - 3 = 0.$$

Que.4(A) Answer any one.

- (1) Define: Right circular cylinder. Find the equation of right circular cylinder with axis (05)
- $$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} \text{ and radius } r.$$
- (2) Find the equation of cylinder whose generator is parallel to the straight line (05)
- $$x = \frac{-y}{2} = \frac{z}{3} \text{ and whose guiding curve is } x^2 + 2y^2 = 1; z = 3.$$

Que.4(B) Answer any one.

- (1) Derive equation of a cylinder of which generator remain parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and passing through a guiding curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0; z = 0$. (05)
- (2) Find equation of right circular cylinder passing through $(1, -2, 3)$ and axis is $\{(2, -3, 5 + k)/k \in R\}$. (05)

Que.5(A) Answer any one.

- (1) Show that $\frac{x}{1+x} < \log(1+x) < x, x > 0$. (05)
- (2) Evaluate: (i) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\log \tan x}{1 - \tan x}$ (03)
- (ii) $\lim_{x \rightarrow 0} \frac{a^x - 1}{b^x - 1}; a, b > 0$. (02)

Que.5(B) Answer any one.

- (1) Find the co-ordinates of the points where the line $\frac{x-1}{2} = \frac{y-6}{3} = \frac{z-4}{4}$ intersects the sphere $x^2 + y^2 + z^2 = 10$. (05)
- (2) Find the equation of cylinder whose generator is parallel to X-axis and touching the sphere having center $(1, 2, 0)$ and radius 3. (05)
